SCHUNDNER & YOUNG

Exciting Current of Transformers

in Three-Phase Circuits

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EXCITING CURRENT OF TRANSFORMERS

IN THREE-PHASE CIRCUITS

BY

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AND
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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

LEO VINCENT SCHUNDNER AND ROBERT GARDINER YOUNG

ENTITLED EXCITING CURRENT OF TRANSFORMERS

IN THREE-PHASE CIRCUITS

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

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CONTENTS.

INTRODUCTORY.

When a sinusoidal wave of electromotive force is impressed upon a coil with an iron core, the current which flows will not have the form of a sine wave, but will be distorted by the presence of higher harmonics. In three phase circuits this wave distortion has an extremely important bearing on transformer connections.

In this investigation, higher harmonics of frequencies as high as the eleventh were observed, but the third proved by far the most important. Those harmonics above the fifth were found to be relatively unimportant and their maximum values so small as to be practically negligible.

Molecular friction - i.e., magnetic hysteresis, is the more important source of losses in an iron-clad alternating current circuit. A wave of current produces an alternating magnetic flux which induces a counter E.M.F. of self induction. Neglecting any ohmic resistance, the counter E.M.F. will equal the impressed and their waves will be similar. If the impressed E.M.F. is a sine wave, the counter E.M.F. will also be a sine wave. Now

$$e_{\text{ind.}} = - n \frac{d\Phi}{dt}$$

or

$$n \, d\Phi = - e_{\text{ind.}} \, dt$$

Then,

$$n\Phi = - \int_{T_1}^{T_2} e_{\text{ind.}} \, dt$$

that is, the magnetic flux which generates the induced E.M.F. must follow the form of a cosine wave, or a sine wave displaced

90^0 from the $e_{ind.}$ wave. The alternating current wave will not be a sine wave, but will be distorted by the presence of higher harmonics and a complex wave will result. This distortion is due to the magnetizing current. It is caused by the disproportionality between magnetic induction and magnetizing force - shown by the magnetization curve, and the effect of hysteresis is negligibly small.

The amount of distortion of the current wave depends upon the density of the flux. The effect of a change of density is readily seen. As the density in the magnetic path approaches saturation an increasing value of magnetizing current must be supplied for each increment of magnetization obtained. A portion of the exciting current supplied is utilized as hysteresis power current as it is in phase with the impressed E.M.F. and is practically a sine wave. The magnetizing current is wattless and tends toward a peaked wave form, the more nearly saturation is approached. The higher the value of the magnetizing current, the greater the disproportionality between the magnetic induction and the magnetizing force, the greater the distortion of the current wave and the higher the maximum values of the harmonics.

The E.M.F.'s, and currents in a three phase system are displaced, one from the other, by 120^0. The third harmonics differ in phase by 3×120^0 or 360^0 (a complete period) and are therefore in phase with each other. From this consideration it is evident that wherever third harmonics appear in a three phase system, the system is single phase, for them. In the star-delta

connection of transformers, it is found that a reading of voltage obtained across one corner of the opened delta is, in magnitude, three times the third harmonic. This is due to the fact, that, the observed voltage is the sum of three triple harmonics in phase with each other. This result would be expected, in view of the statement just previous.

The sum of the three E.M.F.'s. between lines of a three phase system -delta voltages- is zero. As the third harmonics are in phase with each other and are cumulative, it follows that the voltages between the lines of a three phase system cannot contain any third harmonic or its overtones.

As the sum of the three currents in a three phase system is zero, from the consideration that their third harmonics are in phase with each other, it is evident that the currents in the lines of a three phase, three wire system (the Y currents) cannot contain a third harmonic or its overtones.

Third harmonics can, however, exist in voltage between line and neutral of a star system or in currents between lines of a delta system. Then, in the star system, as the third harmonics are in phase with each other, a potential difference of triple frequency exists between the neutral and any line: the whole system pulsating against the neutral at triple frequency. In the delta system, since the coil currents have a phase of 60°, their third harmonics are in opposition and, therefore, neutralize each other. Hence, the third harmonics in delta currents of a three phase system do not exist in the line but only in the coil.

APPARATUS.

The sine wave of impressed E.M.F. was taken from the generator end of a motor-generator set - 3 phase, 60 cycles.

Oscillograms were taken from a General Electric Oscillograph and transferred to coordinate paper by means of a General Electric wave Micrometer.

Ammeters and voltmeters used in the alternating current circuits were General Electric, portable type, movable-coil instruments. Direct current measurements for calibration were taken with Weston ammeters and voltmeters, also of the portable type, movable-coil pattern.

Wattmeters used were the General Electric, portable type, non-compensating.

All meters were calibrated and corrected from curves.

Three, single phase, 60 cycle, 1.5 K.W., 2200/1100 volt primaries, 220/110 volt secondary, core type, transformes were used in this investigation. The low voltage coils only were used, giving a one to one ratio of transformation. The high voltage coils were open circuited and all external taps disconnected.

Diagrams for the interconnection of the transformers are shown in Fig.I.

As a basis for discussion and as a representative commercial type, a core type transformer was selected, on which oscillograms of exciting current were taken with a sinusoidal applied potential (See Curve No. I). The oscillogram of exciting current and applied E.M.F. at normal density is shown in Curve No. 2 and

same for density above normal in Curve No. 3. The applied E.M.F. and the flux waves are practically sine waves and are assumed as such. The analysis of the waves of exciting current show the distortion due to higher harmonics.

The complex current waves reach their maximum values at the same time with the flux wave. This necessarily follows, as maximum current is obtained when the flux is a maximum, or when the greatest number of lines of force are cut in time dt..

Inspection of the waves leads to the following interesting results. The current wave is bulged out on the increasing and hollowed out at the decreasing side. With increasing density or higher magnetization, the maximum of the current wave becomes more peaked and the wave becomes more flattened at the zero points. The maximum values of the higher harmonics increase with the density.

At both densities a strong third harmonic is shown. At normal denisty the harmonics above the third are negligible, but at the higher density the fifth and seventh harmonics are also of importance while the ninth and eleventh are appreciable.

Im this connection it might be said, that, in a magnetic circuit in which power is expended aside from hysteresis, the distortion of the wave of exciting current is not as large as in the case of a transformer on open circuit (as in this investigation). As soon as the system expends energy in any other way, the energy compenent of the current increases greatly, and though the distorting component has the same maximum value, its per cent

value of the total current is decreased as the total current increases, until the distorting effect is almost nullified. On a system where transformers are operating on low load-factor, a knowledge of the distorting effect of the magnetizing current might prove useful. An interesting investigation could be readily conducted by closing the transformer secondaries through low resistances or inductances and varying the load over such ranges as would give the magnetizing current an appreciable per cent value.

In connection with Curves Nos. 2 and 3 the following operations tions were performed: (1) Using the complex wave of current and the sine wave of flux a hysteresis loop was plotted, the area of which represents the energy given to the core for the conditions considered. (2) Using the fundamental component of the complex current wave (the complex wave minus the higher harmonics) and the sine wave of flux, an elliptical hysteresis loop is produced, of the same area as the distorted loop. This leads to the conclusion that, with sinusoidal applied potential, the higher harmonics in the current wave neither are produced by, nor produce, the energy loss in the core.

The general conclusions to be drawn from the above single phase tests and curves are: (1) In transformers the distortion of the wave of exciting current is neither a cause nor an effect of the energy loss in the core, but results only from the varying permeability of the steel. (2) The amount of distortion varies with the density as do the amplitudes of the higher harmonics.

For a given effective value of E.M.F., the wave may have a great variety of shapes. Since $e = -n\frac{d\Phi}{dt}$ nd $\Phi = -\int_{T_1}^{T_2} e\, dT$, the shape of the flux wave depends upon the shape of the E.M.F. wave: The more peaked the latter, the less area it encloses and the lower the value of $\int_{T_1}^{T_2} e\, dT$ or vice versa.

The maximum value of the flux wave depends upon the maximum of the integral e dt and so varies with the area enclosed by the E.M.F. wave; The less the area the lower the maximum value of the flux wave.

$P_h = K_h f B_m^{1.6}$, where P_h = hysteresis loss in watts., f is the frequency in cycles per second, B_m is the maximum flux density and K_h is a constant, depending upon the quailty of iron. $B_m = \frac{\Phi_m}{\text{Area}}$, so B varies as Φ and so varies the hysteresis loss, P_h, in direct ratio.

Sheet 1 gives core loss for star open delta connection where the coil voltage waves were distorted by the presence of a triple harmonic, giving the peaked wave shown in figure 7. Sheet 1 also gives losses for the transformer connected single phase, with sine wave of E.M.F. (as shown in Curve #1) impressed. An examination shows that for the same effective value of E.M.F. for the two connections, the peaked shaped E.M.F. wave gives the lower core loss by a considerable per cent, this confirming the view expressed just previously.

INTERCONNECTED TRANSFORMERS

For the investigation of exciting currents in three phase circuits, three single phase transformers were used similar to the one selected for the single phase test. With three transformers, the star-delta connection is without doubt, the most important and will be considered first (See Fig. 1) Oscillograms of current and voltage for this connection are shown in Curves No. 4 - No. 8 inclusive.

With a sine wave of E.M.F. impressed upon the terminals of a transformer, the exciting current must contain a strong third harmonic. Third harmonics of current cannot exist in lines of three phase system. Consideration of a star connection of the primaries of three transformers- secondaries open circuited- shows the following relations:

(1) Instead of the voltage of each transformer being $\frac{Eo}{\sqrt{3}}$ and a sine wave, it must contain (a) a triple harmonic and its overtones but (b) no other harmonics. If the wave did not contain a third harmonic, the exciting current taken by the transformer would contain a third harmonic. But such a third harmonic cannot exist in a three phase system. Therefore the wave of magnetism must be a complex wave, containing a third harmonic about opposite to that which is suppressed in the exciting current. If on the other hand, the wave contained other harmonics than the third or its overtones, they would not be eliminated between lines and the impressed Eo would not be a sine wave. E(coil) will then have an effective value E, which equals $\frac{Eo}{\sqrt{3}}$ increased by the

effective value of the triple harmonic present between line and neutral.

(2) The exciting current in the transformers cannot contain any third harmonic or its overtones, but may contain any other harmonics.

(3) The magnetic flux is not a sine wave but contains a third harmonic and its overtones similar to those of the voltage wave.

Curve No. 7 bears out these conditions as regards voltage. The maximum of the third harmonic is 59% of the fundamental maximum and 38% of the complex wave maximum. The ninth harmonic has a maximum equal to 4.5% of the complex wave maximum. Others are negligible.

Curve No. 4 is for exciting current in a star-delta system and shows an appreciable fifth, seventh and eleventh harmonic. The third and ninth are negligible and their presence is accounted for by the fact that the E.M.F. impressed is not a true sine wave, as was assumed.

In the star-delta connection the third harmonics of E.M.F. generated in the transformer secondaries are in series in short circuit and produce a local circulating current in the secondary circuit. This current is of triple frequency and supplies the third harmonic of exciting current which was suppressed in the primary because of the star connection. This triple circulating current eliminates the third harmonic of magnetism and of E.M.F. It could not exist in the primary by virtue of the star connection so that its presence in the secondary is a necessity.

A consideration of the actual coil conditions shows that

each coil in the primary has a voltage wave distorted by the presence of a third harmonic, and this harmonic appears in the voltage wave of each secondary coil. As the secondary is connected in delta, the sum of the complex waves of voltage would be zero, were it not for the triple frequency voltages in each coil. As these voltages are in phase with each other and are cumulative, it is the sum of the triples (or three times the triple voltage wave) which is read when the drop across the opened corner of a delta is taken. If the delta be closed, a triple frequency circulating current, due to this triple voltage will flow, its effect being to smooth out the distortion in the primary voltage wave as explained previously.

Thus, by connecting the secondaries in delta, the wave distortion disappears, the waves of E.M.F. and flux become sine waves and the exciting current becomes a wave of form corresponding to a sine wave of flux but is distributed between primary and secondary: The third harmonic is produced by self induction in the secondary but does not appear in the primary.

Curves Nos. 2 & 3 show exciting current waves for sine waves of flux in a single phase circuit. From the above it would be expected that a wave of exciting current for a star system of connection would differ from waves taken from a single phase connection only in the abscence of the triple harmonic which appears in the secondary of the latter. Curve No. 5 shows primary star current and circulating current in the delta for star-delta connection. These waves were added algebraically and the resultant wave (Curve No 5 A) plotted. This wave is similar to

waves Nos. 2 and 3 and possesses the characteristics in an earlier portion of this paper noted as peculiar to a wave of exciting current in an iron-clad alternating current circuit, with sinusoidal impressed E.M.F.

Curve No. 8 shows the circulating triple current in a star-delta system, a higher density being used than in the case of Curve No. 5. Analysis of this wave shows the presence of a fundamental with maximum about 14% of the triple maximum, and a trace of a fifth harmonic. The wave is however, almost pure triple frequency.

The presence of other harmonics than the third in this connection is due to the fact that the transformers were not well balanced. The amplitudes of the respective waves are not of equal magnitude, because of this lack of uniformtiy, and therefore do not give the results obtainable if the transformers were balanced. As shown in the discussion of delta currents for balanced conditions, the circulating current in this connection is purely triple.

Curve No. 6 is for star - open delta. As the third harmonic exciting current cannot be supplied, the flux wave must contain a third harmonic. This third must oppose and equal the M.M.F. supplied by a third harmonic which would be required by a semi-soidal wave of flux. The coil E.M.F. will then contain a corresponding third harmonic. Other harmonics than the third and its overtones will appear in the exciting current as in any three phase system. A voltage reading across the open delta will equal the value of one fundamental plus two triple voltages.

This is evident when the secondary connection is considered. In the two coils the voltages are displaced 120°. The triple frequency voltages are in phase with each other, so two times E triple is obtained. The sum of the voltages of a three phase system is zero, as the voltages contain no triple harmonics. Therefore, the sum of two voltages equals the third, the phase relation of the three waves being 120° with each other. Hence, a voltage reading across the open delta will have the value of one fundamental and two triple voltages.

An observation of the various voltage waves taken shows a tendency toward peakedness in each instance. This distorting effect of the harmonics has then a tendency to increase the maximum voltage of the system and therefore the strain on the insulation. The higher the potential, the greater is the danger due to this distortion. The maximum value of voltage is the puncturing value and the reliability of operation of a transformer may be greatly reduced by the increase in potential strains. In large ? transformers, where the insulation factor is low, -about two- this is an important consideration. Strains resulting from potential distortion must be distinguished from those due to to surges or resonant phenomena, as the former are distributed equally over all parts of the winding, while the latter are localized or concentrated on some portion of the winding.

Referring to Sheet 1, which is for star - opened delta connection, it is seen that $E_{line}/\sqrt{3}$ is less than E (coil) in each case, varying from the values shown for E_A by 9.3% in the case of the highest. The lowest voltage is 82% of normal, the highest is 145% of normal and for this range, the increased effective value, due to the triple frequency voltage in the coil varies from 9% to 14%. The maximum value varies somewhat in the same proportion but is dependent upon the wave shape. For ordinary ranges of voltage, an increase of about 10% in coil E.M.F. may be expected, due to the triple harmonic. In a high voltage system it is evident that the increase of insulation strain, from this source, may be of serious moment.

The values and per cents as calculated are from comparison with coil A or voltage E_A. The condition under which these readings were taken are met with in operating, either in star-open-delta or in star - star, and so are of prime importance. If the secondary deltas were closed the circulating triple would, of course, supply the triple harmonic of flux necessary, and the distortion of the coil voltage wave would vanish.

DELTA-DELTA CONNECTION.

Curves No. 4, 10, 11, are plotted from oscillograms taken from the same transformers with both primaries and secondaries in delta.

Curve No. 9, shows secondary delta circulating current, which is approximately a wave of pure triple frequency. The maximum of the fundamental is about 12 % of the maximum of the complex wave. This current is produced by the triple, existing between lines in the primary circuit as was explained previously.

Curve No. 10. shows line current wave for a delta-open-delta connection. The complex wave is composed of a large fundamental, with fifth and seventh harmonics of appreciable value, but with no third.

Curve No. 11, shows the wave for primary coil current for a delta-open-delta connection. As would be expected, a large third harmonic is present, in addition to other higher harmonics. The maximum of the triple is about 52 % of the fundamental maximum and about 30 % of the complex wave maximum. As the density in this case was slightly below normal, the percent value of the triple harmonic is a good indication of the importance of harmonics in general as factors in transformers practice. The value of knowledge of the harmonic relations cannot be overvalued in the operation of a transformer system.

CONCLUSIONS.

The general conclusions to be drawn from these tests and curves are:

1. In transformers, the distortion of the exciting current wave shape is neither a cause nor an effect of the energy loss of the core but results from the varying permeability of the field.

2. The amount of the distortion varies with the density of the megnetic flux, and the higher the density, the more prominent are the higher harmonies.

3. A strong triple frequency wave of current or voltage is present in some portion of each connection.

4. The amplitude of the third harmonic increases with the density, its maximum value depending to a certain extent upon the relative value of other harmonics present in the current wave.

5. With sinusoidal applied potential, the higher harmonics in current wave are not produced by or do not produce the energy loss in the core.

6. In a star-delta system, with sinusoidal applied potential.

a. The voltage from coil toneutral will contain a triple harmonic and its overtones but no other harmonics.

b. The exciting current cannot contain any third harmonic or its overtones but may contain any other harmonics.

c. The triple frequency component of exciting current necessary for a sinusoidal impressed E. M. F. wave is present as a circulating current in the secondary.

d. If the secondary is open delta so that no circulating triple current is present, the flux wave must contain a third harmonic.

7. In a delta-delta system, with sinusoidal impressed E.M.F.

a. The line current is a complex wave but contains no third harmonic or its overtones.

b. The exciting current eontains a large triple harmonic as well as other harmonics.

c. In the secondary delta a circulating triple current exists, which is induced by the triple frequency current in the primary.

8. The core loss depends upon the shape of the E.M.F.wave.

9. Distorted potential wave shapes, increase the potential strains throughout the transformer winging and reduce the reliability of operation.

APPENDIX.

A simple mathematical proof (due to Thompson) of the observed fact that the core loss is independent of the higher harmonics is submitted:

The area of the hysteresis loop represents the energy expended in a complete cycle of magnetizing operations. Representing the impressed E.M.F. as a sine function of the time, the wave form of the flux will be a pure cosine function and the reactive E.M.F. a pure negative sine function of time. However, if there are present hytersis and eddy currents - as always in an iron clad circuit -the current wave will not have a simple wave form but may contain terms of the following orders:

$$A_1 \sin \omega t$$
$$B_1 \cos \omega t$$
$$A_3 \sin 3\omega t$$
$$B_3 \cos 3\omega t$$
$$A_5 \sin 5\omega t$$
$$B_5 \cos 5\omega t$$

and possibly higher orders. The impressed voltage form is expressed by E $\sin\omega t$ and the total energy for a cycle can be represented by multiplying E $\sin \omega t$ into each of the above terms and intergrating about a whole cycle - i,e. from $t = 0^\circ$ to $T = 360^\circ$.

But the following integrals all reduce to zero if integrated over a whole period.:

$$\int \sin \omega t \cos \omega t$$

$$\int \sin \omega t \quad \cos \omega t$$
$$\int \sin \omega t \quad \sin \; n \omega t$$
$$\int \sin \omega t \quad \cos \; n \omega t$$

Then, the only product remaining is the fundamental :

$$\int_0^T \sin^2 \omega \; t$$

In words, this means that the only component which involves the expenditure of any energy in the cycle, is the sine conponent of the first order. All other components may distort the loop, but will not vary its area in the least.

In all cases the area of the loop is equivalent to an ellipse, the principal axes of which are respectively, the maximum flux density, and the value of $\mathcal{H}$ due to the maximum value of the first sine component of the complex current curve.

Analysis.

A method of analysis by groups, due to Professor C. Runge (Zertschrift für Mathematik und Physik XLVIII P.443 - 1903) condensed and revised by Professor S. P. Thompson in Vol.II of his work on Dyamo Electric Machinery, was used in the analysis of the waves in this article.

Any univalent periodic function may be expressed by the series,

$$Y = a_0 \quad a_1 \cos \theta \quad a_2 \cos 2\,\theta \quad a_3 \cos 3\,\theta \; .... \; a_n \cos n\,\theta$$
$$b_1 \sin \theta \quad b_2 \sin 2\,\theta \quad b_3 \sin 3\,\theta \; .... \; b_n \sin n\,\theta$$

where the coefficients a_1, a_2, ... b_1, b_2, ... do not involve θ

As even harmonics are not present in alternating

waves, their effective value being zero for a period, only odd harmonics need be considered. Rewriting the equation and disregarding the even harmonics the following equation is obtained.

$$2) = a_1 \cos \theta + a_3 \cos 3\theta + a_5 \cos 5\theta \ldots + b_1 \sin \theta + b_3 \sin 3\theta + b_5 \sin 5\theta \ldots$$

It is assumed; that the mean horizontal axis is midway between the highest and lowest points of the wave; that the zero point of the wave is chosen where the wave crosses this axis; and that the position and negative portions of the waves are idenical so that only a half wave - 180° - need be considered. The unknown coefficients $a_1, a_2, a_3 \ldots b_1, b_2, b_3 \ldots$ must be found. These will then be combined to determine $R_1, R_2, R_3 \ldots$ according to the equation

$$R = \sqrt{a^2 + b^2}$$

and the angles $\Phi_1, \Phi_3, \Phi_5, \cdots$ by the equation

$$\Phi_1 = \tan^{-1} \frac{b1}{a_1}$$

Where R_1, R_3 etc are the amplitudes of the harmomics, and Φ_1, Φ_2 etc. are the angles of lag with respect to the complex wave. When these are determined the first equation will appear in the form.

$$Y = R \cos (\theta - \Phi_1) \quad R_3 \cos (3\theta - \Phi_3) \quad R_5 \cos (5\theta - \Phi_5)$$

Professor Runge's scheme was to divide the period into 4 n parts since in one period of any sine wave there corresponds four values, one in each quadrant, + or - , of the same sine value for every angle. These four were grouped together for a single multiplication. To separate the odd and even orders of harmonics

second sums and differences were required. The final result was to form a schedule for the various stages and by this method to obtain a complete analysis by a few operations. Professor Runge's schedule was worked out to determine the Fourier coefficients, odd and even to the 18th harmonic. Professor Thompson revised and shortened the schedule using one quarter of the ordinates used by Professor Runge, and decreasing the number of operations. This latter schedule applies only when odd terms to the 11th harmonic are desired. By this method the half period is divided into 12 parts by equidistant ordinates, ordinates being drawn every 15°; The value of the twelfth ordinate is zero. The eleven ordinates are measured and denoted as Y_1 Y_2..... Y_{11}. The ordinates are then arranged in the following manner:

	Y_1 Y_2 Y_3 Y_4 Y_5 Y_6	
	Y_{11} Y_{10} Y_9 Y_8 Y_7	Where S_1 = sum of Y_1 and Y_{11} and
Adding	S_1 S_2 S_3 S_4 S_5 S_6	d_1 = difference between Y_1 and Y_{11}
Subtracting	d_1 d_2 d_3 d_4 d_5 d_6	and so on.

Values for use in determining the third and ninth harmonic are obtained by grouping the numbers as follows:

$$S_1 + S_3 - S_5 = r_1$$

$$S_2 - S_6 = r_2$$

$$d_1 - d_3 - d_5 = e_1$$

The above numbers are then selected according to the schedule shown below, and entered in the tabulation after having been multiplied by the sine set down in the left hand column.

Schedule for Analysis of Curve Number 4.

	1	2	3	4	5	6
	y_1 = 37.2	y_2 = 54.3	y_3 = 51.5	y_4 = 57.1	y_5 = 80	y_6 = 100
	y_{11} = 31.4	y_{10}= 51.5	y_9 = 51.5	y_8 = 60	y_7 = 80	
Sum	68.6	105.8	103.	117.1	160.	100
Dif.	5.8	2.8	0.	-2.9	0	

$S_1 + S_3 - S_5$

68.6
103.
171.6
- 160
r_1=11.6

$S_2 - S_6$

105.8
-100
r_2 = 5.8

$d_1 - d_3 - d_5$

e_1 = 5.8

Sine Terms.

Sine	1st. & 11th.		3rd. & 9th.		5th. & 7th.	
.262	S_1 = 18				S_5 = 41.4	
.500		S_2 = 52.9				S_2 = 52.9
.707	S_3 =72.9		r_1= 8.2		$-S_3$ = -73	
.866		S_4 =101.5				$-S_4$ = -101.5
.966	S_5=153.5				S_1 = 32.8	
1.00		S_5 = 100		r_2 =5.8		S_6 = 100
1st. Col.	244.4		-16.2		1.2	
2nd. Col.	254.4		5.8		51.4	
Sum	6 A_1 = 498.8		6 A_3 = - 10.4		6 A_5 = 52.6	
Dif.	6 A_{11}= -10		6 A_9 = 22.0		6 A_9 =-50.2	
	A_1 = 83.14		A_3 = - 1.73		A_5 = 8.77	
	A_{11}= - 1.67		A_9 = 3.66		A_7 = -8.37	

Cosine Terms.

Sine	1st. & 11th.	3rd. & 9th.	5th. & 7th.
.262	$d_5 = 0$		$d_1 = 1.52$
.500	$d_4 = -1.45$		$d_4 = -1.45$
.707	$d_3 = 0$	$e_1 = 5.8$	$-d_3 = 0$
.866	$d_2 = 2.42$		$-d_2 = -2.42$
.966	$d_1 = 5.6$		$d_5 = 0$
1.00		$-d_4 = 2.9$	
1st. Col.	.97	2.9	-3.87
2nd. Col.	5.6	5.8	1.52
Sum	$6\ B_1 = 6.57$	$6\ B_3 = 8.7$	$6\ B_5 = -\ 2.35$
Dif.	$6.B_{11} = -\ 4.6$	$6\ B_9 = -2.9$	$6\ B_9 = -\ 5.49$
	$B_1 = 1.1$	$B_3 = 1.45$	$B_5 = -\ .39$
	$B_{11} = -\ .767$	$B_9 = -.48$	$B_9 = -\ .915$

$R_1 = \sqrt{83.14^2 + 1.1^2} = 83.14$ $\tan\Phi_1 = \frac{1.1}{83.14} = .0132$

$R_3 = \sqrt{1.73^2 + 1.45^2} = 2.24$ $\tan\Phi_3 = \frac{1.45}{-1.73} = -.838$

$R_5 = \sqrt{8.77^2 + .39^2} = 8.77$ $\tan\Phi_5 = \frac{-.39}{8.77} = -.0445$

$R_7 = \sqrt{8.37^2 + .915^2} = 8.4$ $\tan\Phi_7 = \frac{-.915}{-8.37} = .109$

$R_9 = \sqrt{3.66^2 + .48^2} = 3.67$ $\tan\Phi_9 = \frac{-.48}{-1.67} = .031$

$R_{11} = \sqrt{1.67^2 + .767^2} = 1.84$ $\tan\Phi_{11} = \frac{-.767}{-1.67} = .46$

$\Phi_1 = 0^\circ - 48' -$ lag

$\Phi_3 = 140^\circ - 0' -$ lag

$\Phi_5 = 2^\circ - 33' -$ lead

$\Phi_7 = 186^\circ - 13' -$ lag

$\Phi_9 = 187^\circ - 30' -$ lag

$\Phi_{11} = 204^\circ - 50' -$ lag

Angle with reference to 0 of Complex Wave.

$\Phi_1 = 0^\circ$ 48' lag

$\Phi_3 = 46^\circ$ 40' lag

$\Phi_5 = 0^\circ$ 30' lead

$\Phi_7 = 26^\circ$ 36 lag.

$\Phi_9 = 20^\circ$ 50 lag.

$\Phi_{11} = 18^\circ$ 59' lag.

R_3, R_9 and R_{11} are negligible.

ACKNOWLEDGEMENTS.

The method of wave analysis used is designated as "Analysis by Grouping of Ordinates" and was evolved by Professor Runge of Hanover in "Zeitschrift für Mathematik und Physik", xlviii, P. 443, 1903. Professor Thompson in his "Dynamo Electric Machinery", Vol. 2, describes this method of analysis and develops a schedule for tabulation which greatly simplifies the solution of simultaneous equations which appear in the calculation. Professor Thompson's method (an adaptation of Runge's) with a few minor changes was used in this investigation.

Valuable information on the subject of wave distortion was found in Steinmetz' "Alternating Current Phenomena" and in a paper by John J. Frank entitled "Observation of Harmonics in Current and Voltage Wave Shapes of Transformers" appearing in "Transactions of the American Institute of Electrical Engineers", Vol. XXIX, Part I, 1910.

The writers take this opportunity to particularly acknowledge the coöperation and assistance (in obtaining the curves and data) of Mr. F.G. Willson of the Department.

SHEET 1.

W_A	W_B	W_c	E_A	E_B	E_c	E_{line}	$\frac{E_{line}}{\sqrt{3}}$	%Variation from E_A
20.0	21.0	15.0	114	113.5	113	180	103.4	9.5
25.1	25.1	22.0	128	128	127	200	115.5	9.75
37.5	37.0	34.0	155	155	153	240	138.5	10.6
31.0	31.0	30.0	144	144	141	220	127.0	11.8
50.0	50.0	48.0	176	177	175	270	156.0	11.9
55.0	55.0	50.0	183	183	181	280	161.5	11.8
64.0	64.0	60.0	192	191	188	290	167.0	13.0
66.0	66.0	60.0	198	199	196	298	172.0	13.15
72.0	72.0	70.0	208	207.5	205	310	179.0	13.9
84.0	84.0	80.0	215	215	212	320	184.5	14.2

SINGLE PHASE.

I	E	W
1.3	120	32.5
2.48	140	45.0
4.06	150	50.0
6.49	160	64.0
8.2	165	72.0
9.1	169	84.0

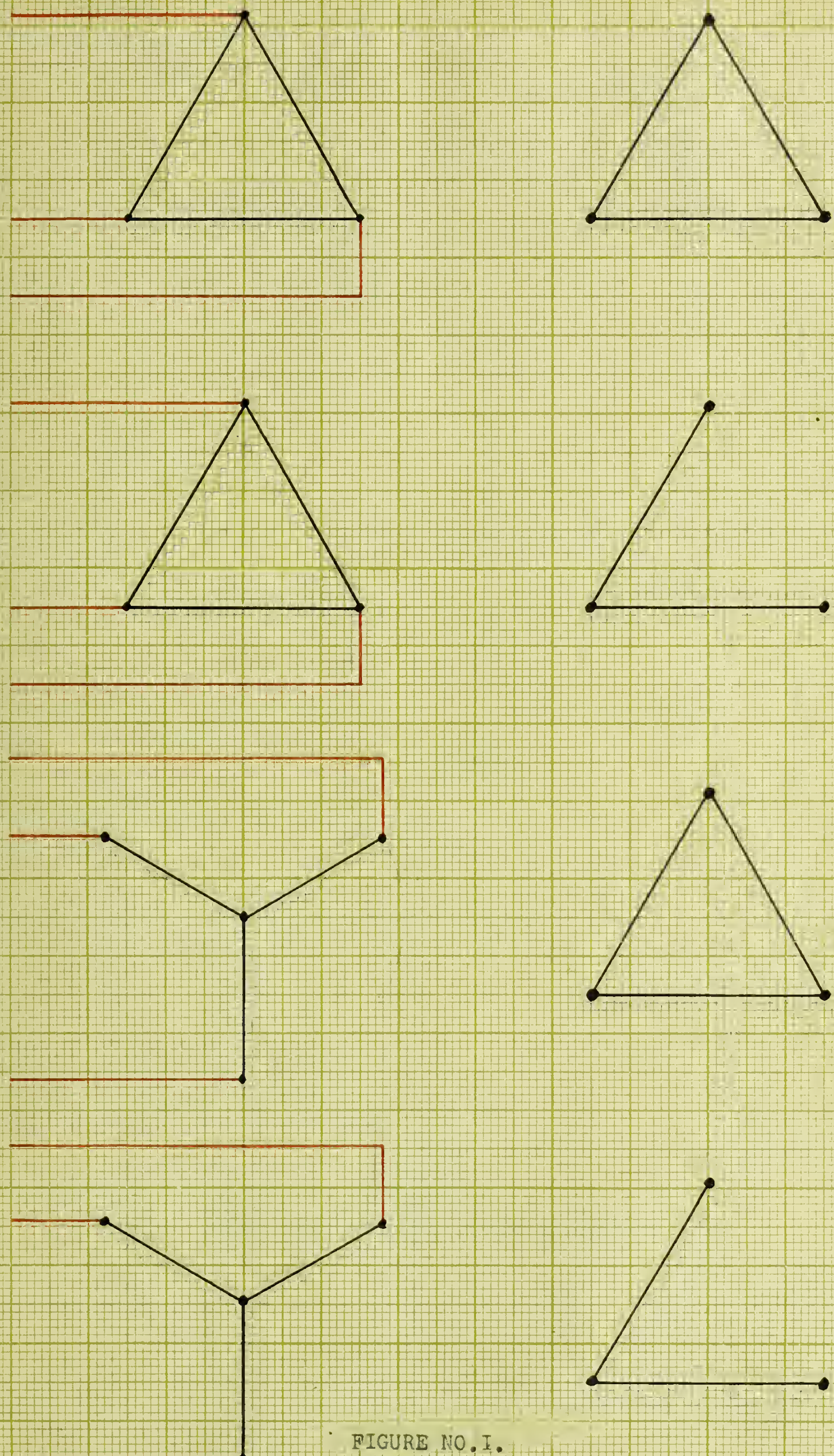
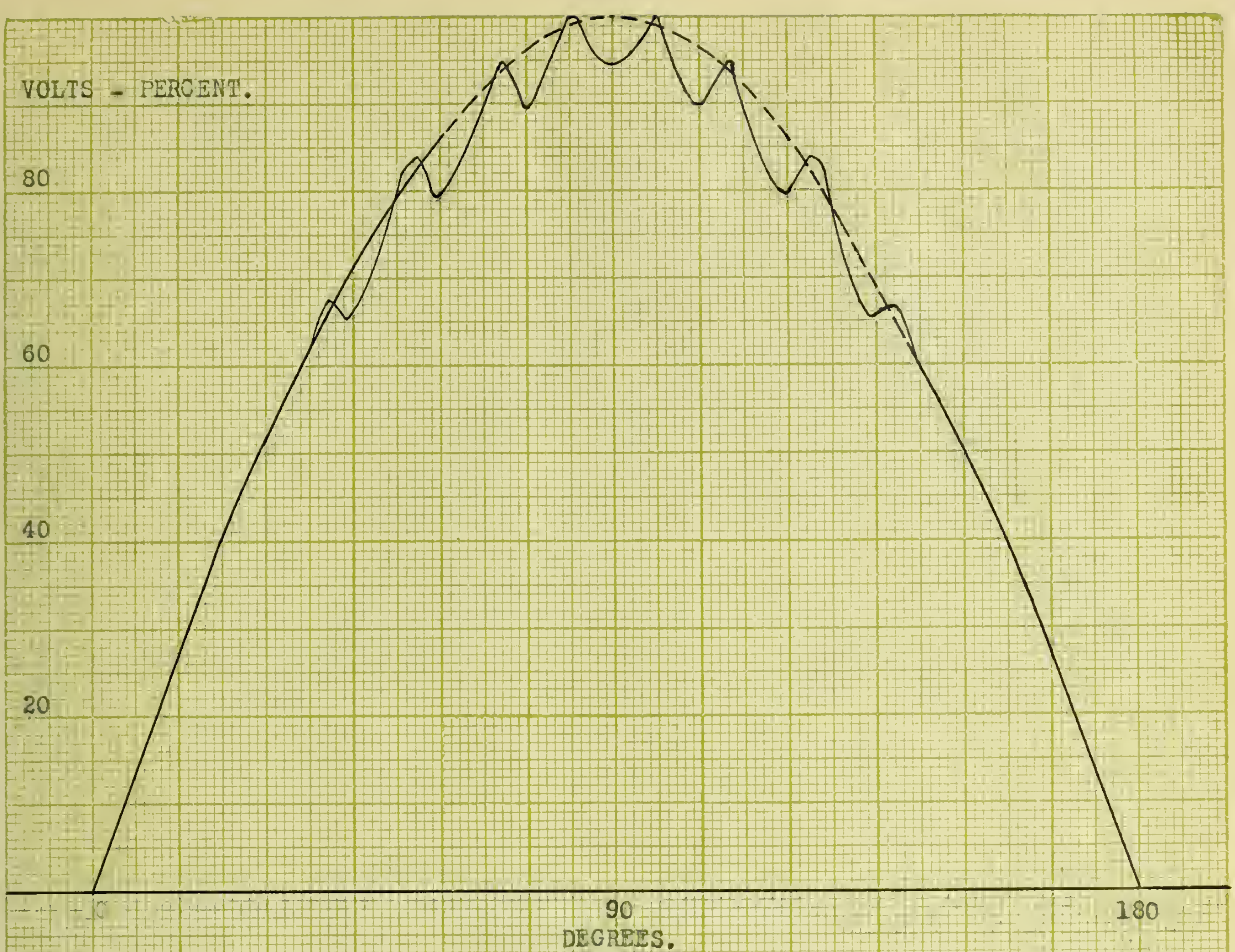


FIGURE NO.I.

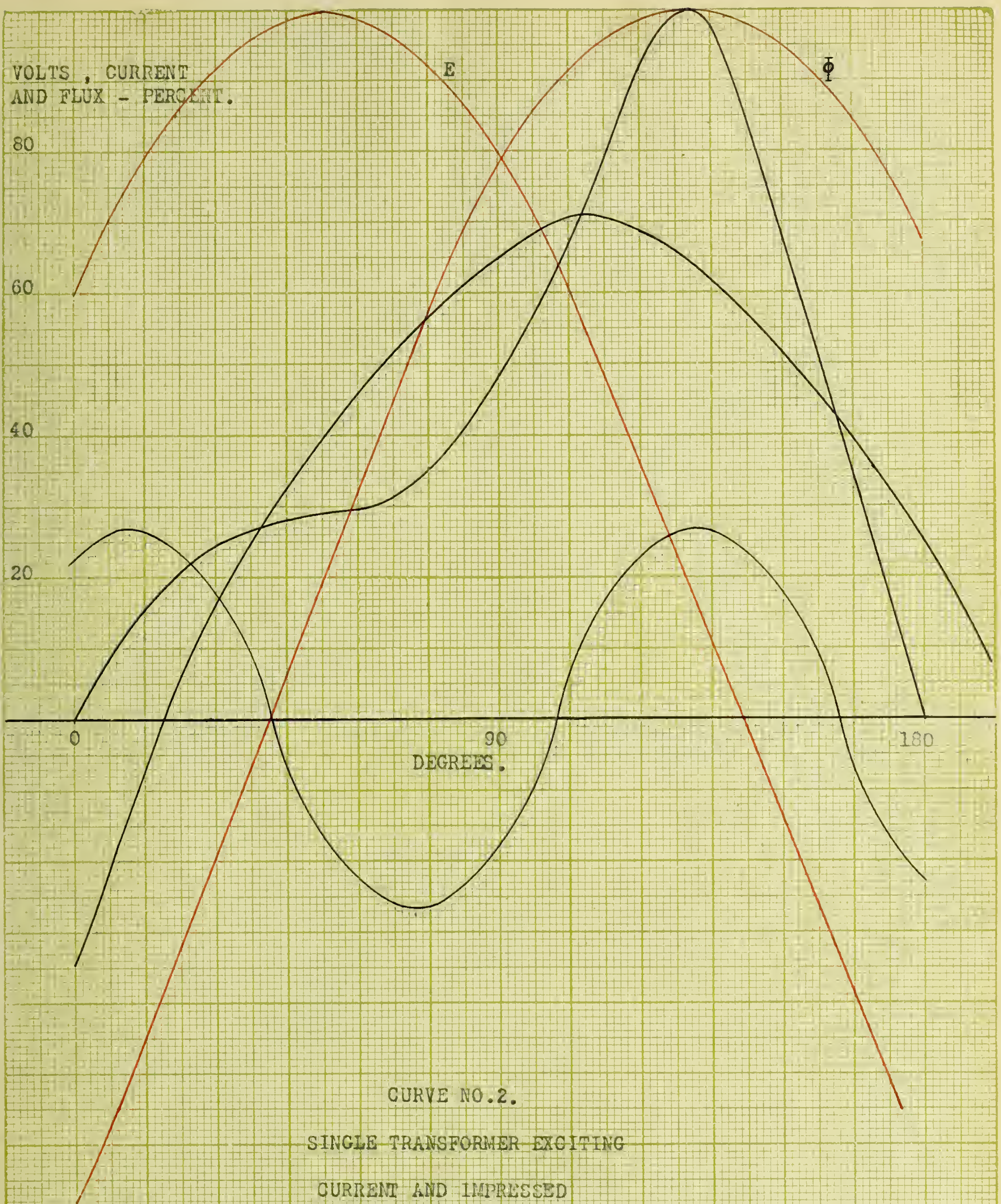
EUGENE DIETZGEN CO., CHICAGO.



CURVE NO. I.

ELECTROMOTIVE FORCE WAVE OF MACHINE.

EUGENE DIETZGEN CO., CHICAGO.



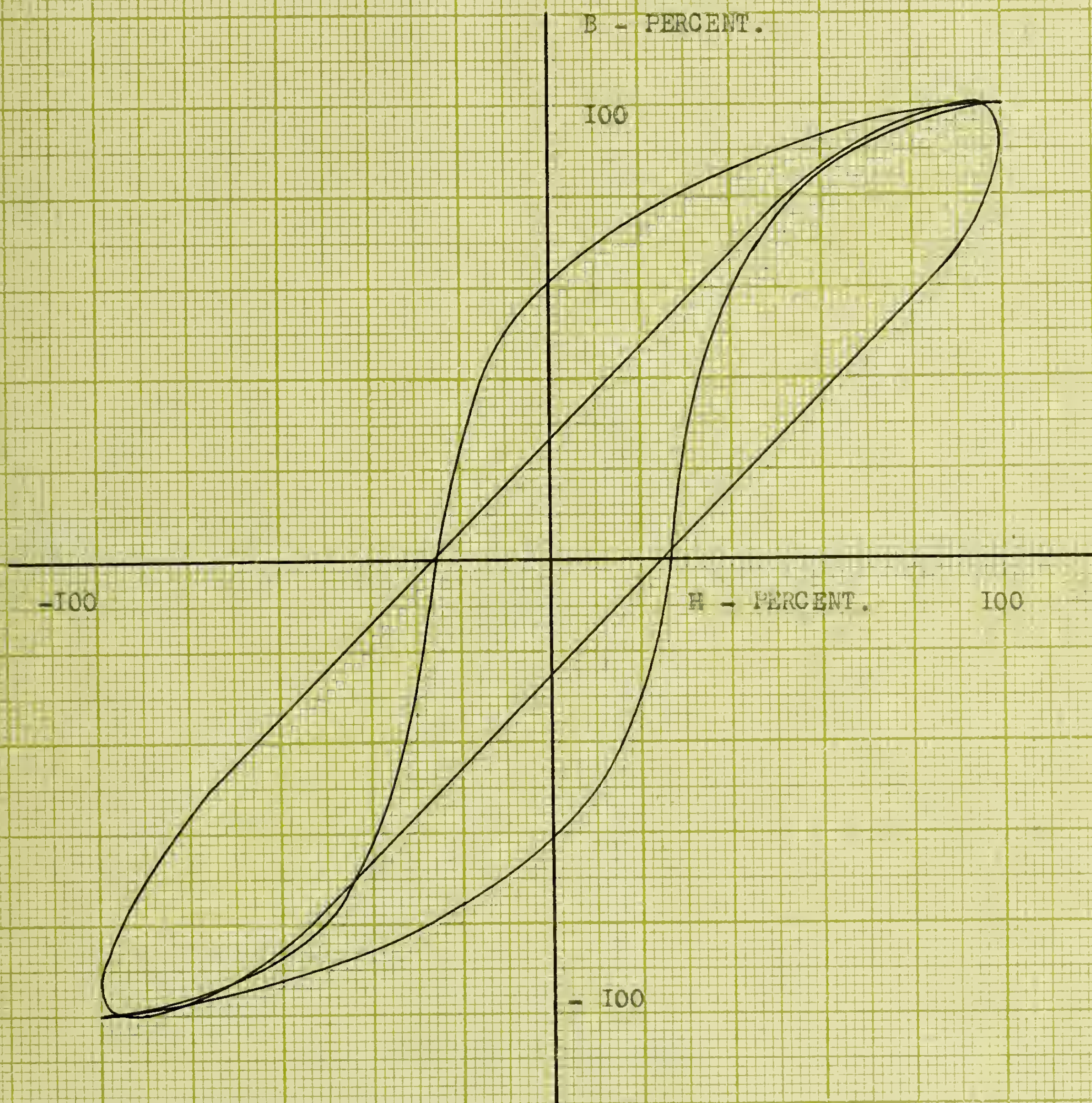
CURVE NO.2.

SINGLE TRANSFORMER EXCITING

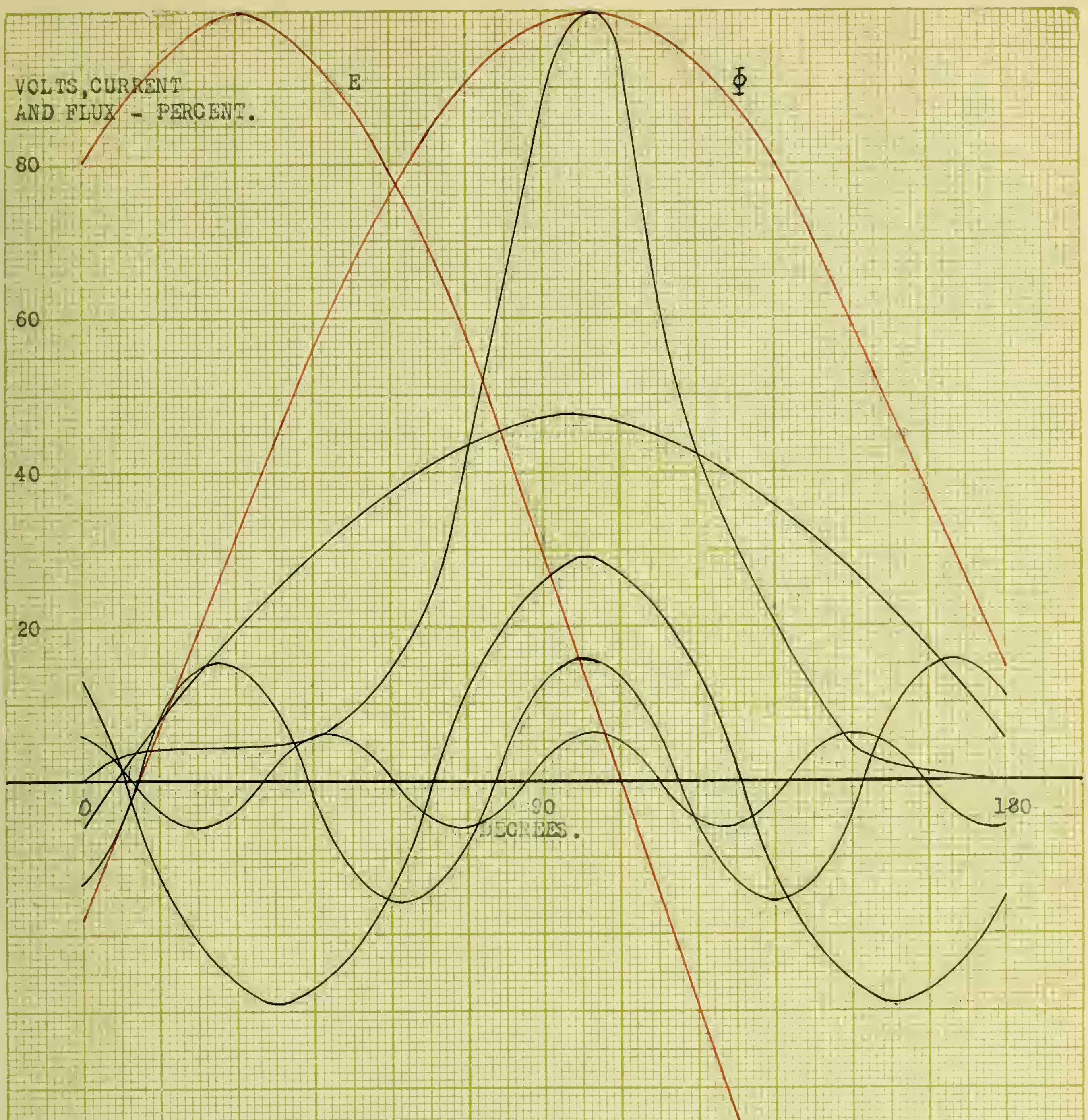
CURRENT AND IMPRESSED

ELECTROMOTIVE FORCE.

R	=	70.8
R^1	=	25.75
Φ_1^3	=	19 25
Φ_3	=	102 30



HYSTERESIS LOOP FOR CURVE NO.2.

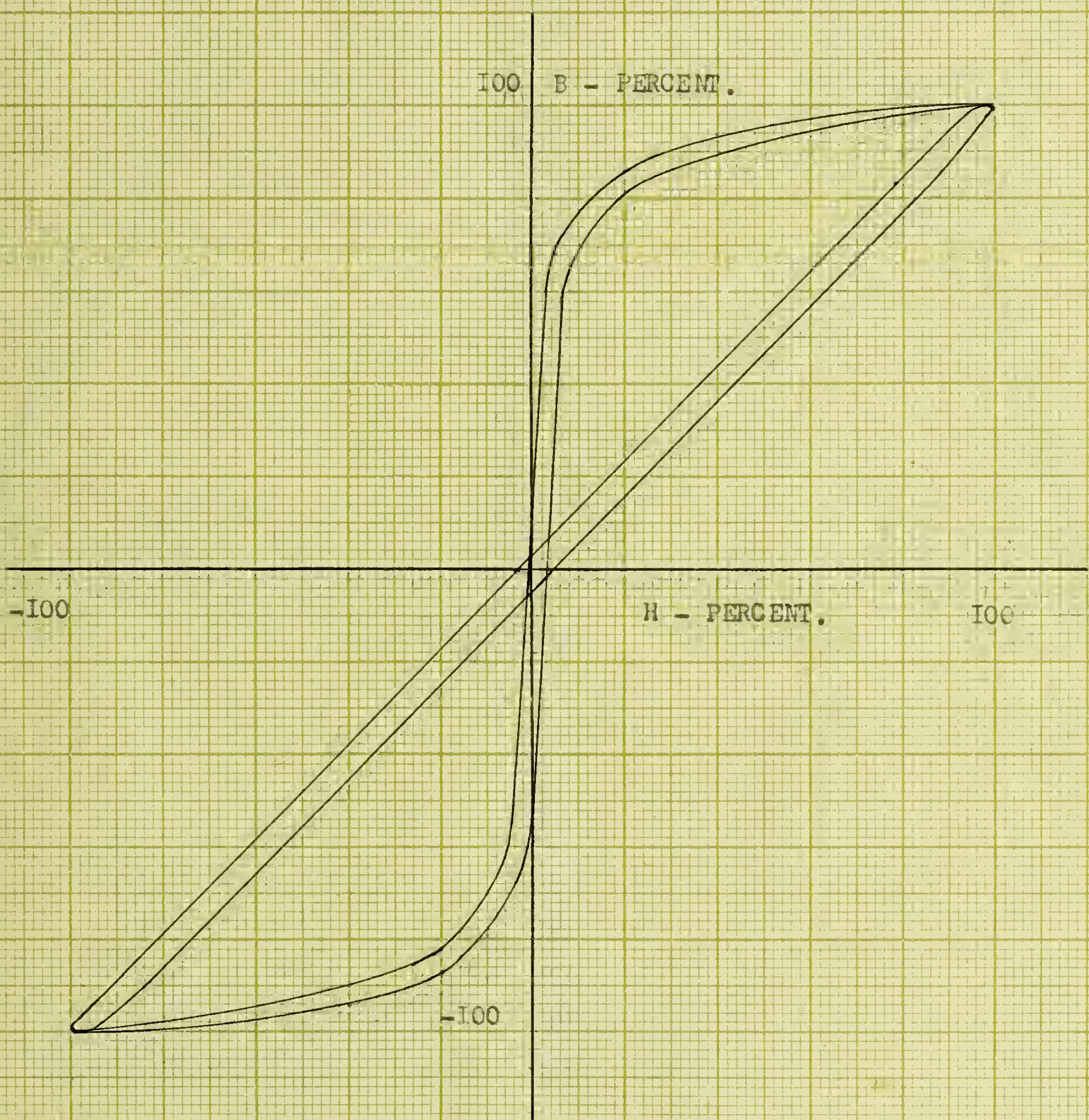


CURVE NO.3.

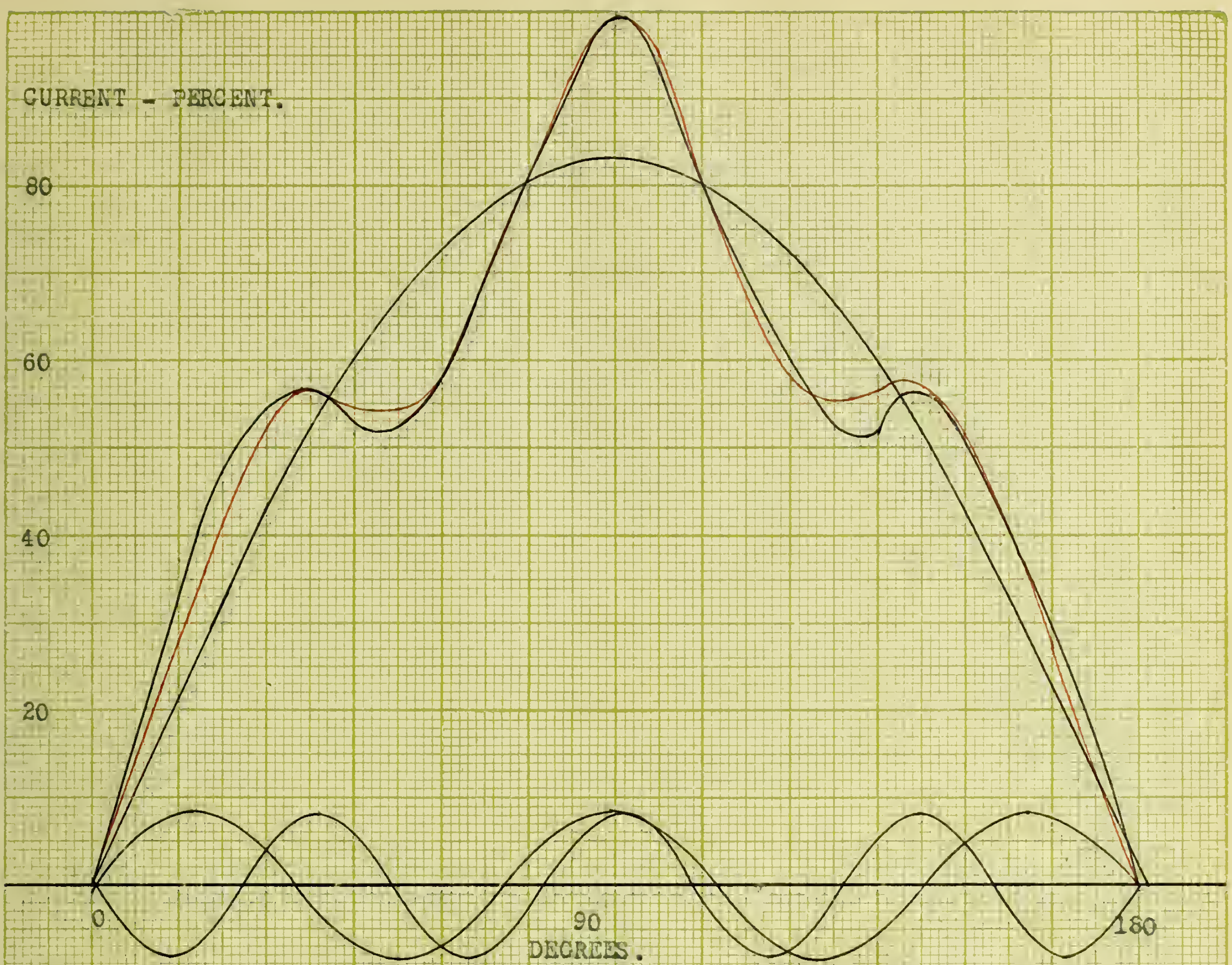
SINGLE TRANSFORMER EXCITING

CURRENT AND IMPRESSED ELECTROMOTIVE FORCE.

$R_1 = 47.6$	$\Phi_1 = 6^\circ\ 0'$
$R_3 = 29.2$	$\Phi_3 = 68^\circ\ 37'$
$R_5 = 15.75$	$\Phi_5 = 8^\circ\ 36'$
$R_7 = 6.15$	$\Phi_7 = 35^\circ\ 45'$



HYSTERESIS LOOP FOR CURVE NO.3.

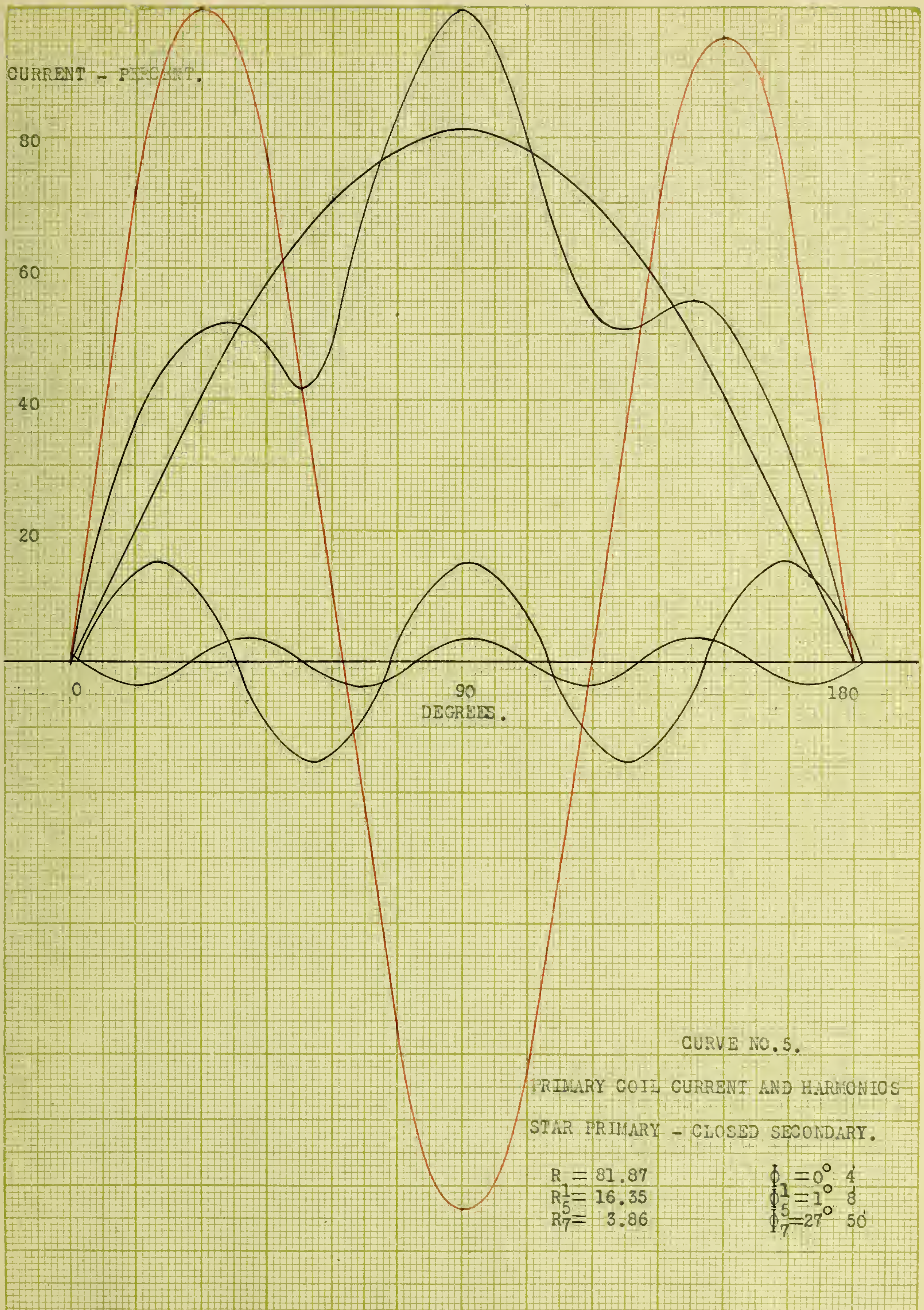


CURVE NO.4.

PRIMARY LINE CURRENT AND HARMONICS.

STAR PRIMARY - CLOSED DELTA SECONDARY.

$R_1 = 83.14$	$\phi_1 = 0^\circ\ 48'$
$R_3 = 2.24$	$\phi_3 = 46^\circ\ 40'$
$R_5 = 8.77$	$\phi_5 = 0^\circ\ 0'$
$R_7 = 8.4$	$\phi_7 = 26^\circ\ 36'$
$R_9 = 3.67$	$\phi_9 = 20^\circ\ 50'$
$R_{11} = 1.84$	$\phi_{11} = 19^\circ\ 0'$



CURVE NO.5.

PRIMARY COIL CURRENT AND HARMONICS

STAR PRIMARY - CLOSED SECONDARY.

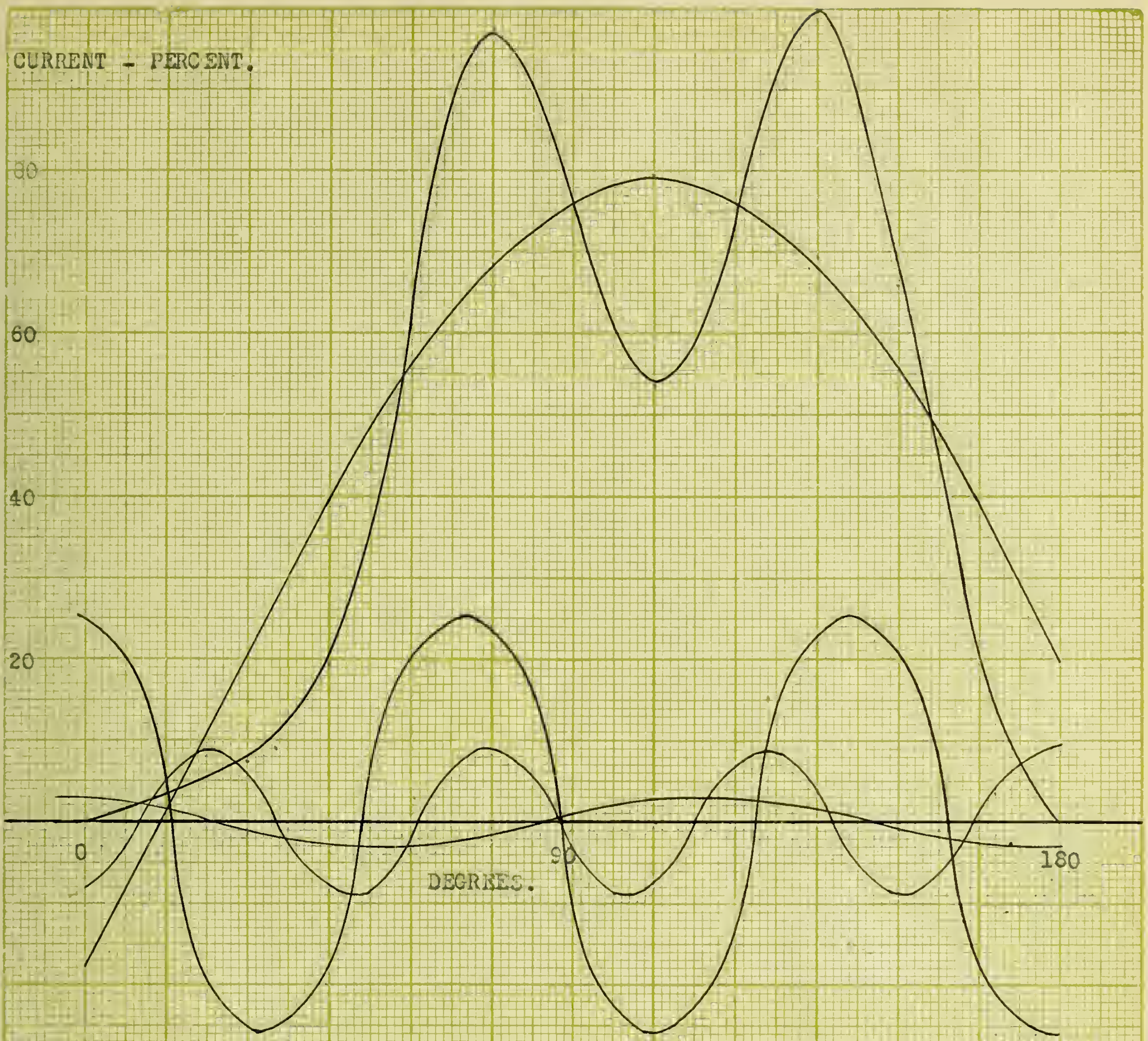
$R_1 = 81.87$ $\qquad$ $\phi_1 = 0^\circ\ 4'$

$R_5 = 16.35$ $\qquad$ $\phi_5 = 1^\circ\ 8'$

$R_7 = 3.86$ $\qquad$ $\phi_7 = 27^\circ\ 50'$

CURVE NO.5a.

EUGENE DIETZGEN CO., CHICAGO.



CURVE NO.6.

PRIMARY COIL CURRENT AND HARMONICS.

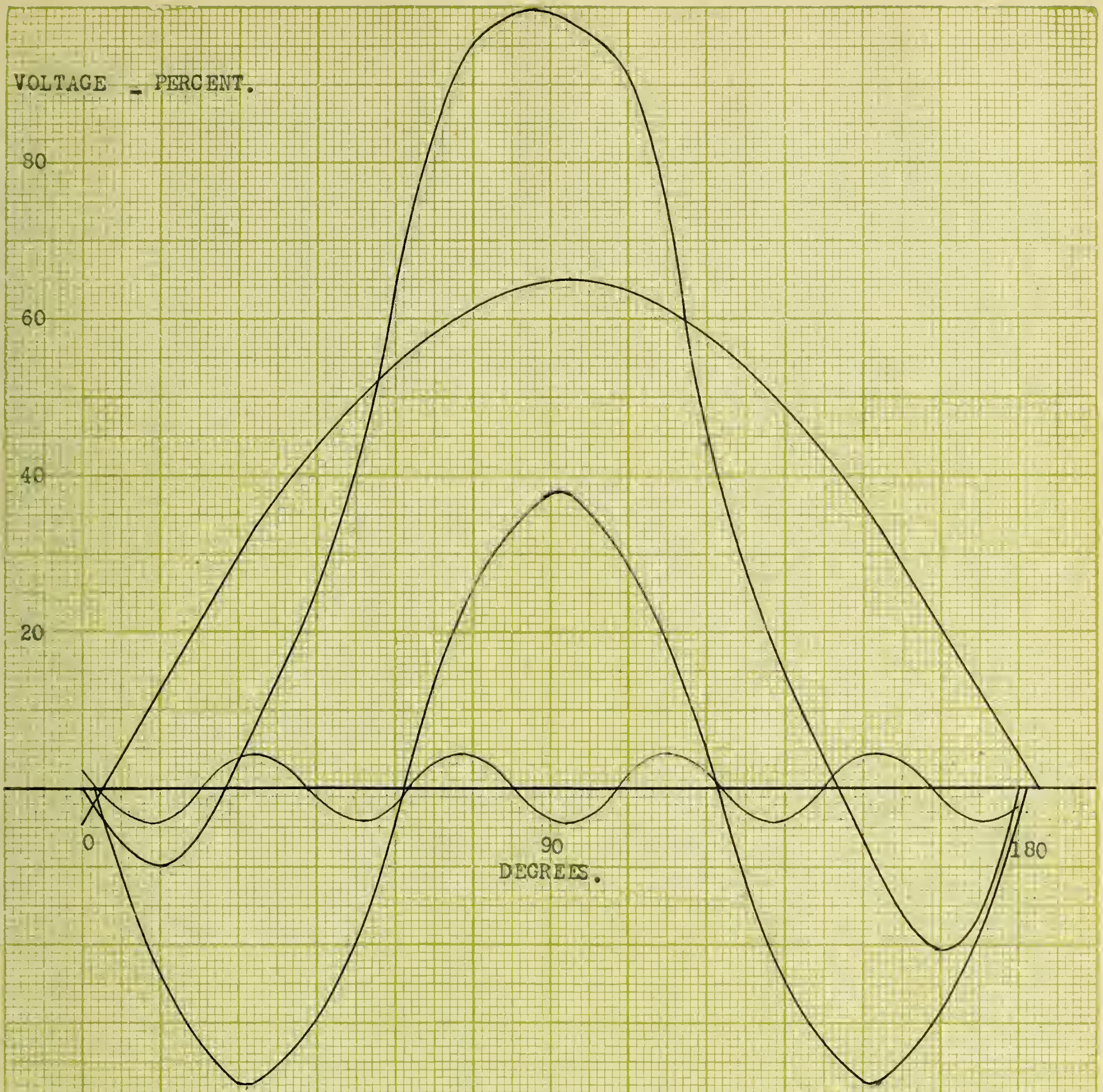
STAR PRIMARY OPEN DELTA SECONDARY.

$R_1 = 80$ $\qquad$ $\Phi_1 = 14^\circ 16'$

$R_3 = 5.14$ $\qquad$ $\Phi_3 = 85^\circ 24'$

$R_5 = 25.6$ $\qquad$ $\Phi_5 = 51^\circ 42'$

$R_7 = 9.0$ $\qquad$ $\Phi_7 = 10^\circ 24'$

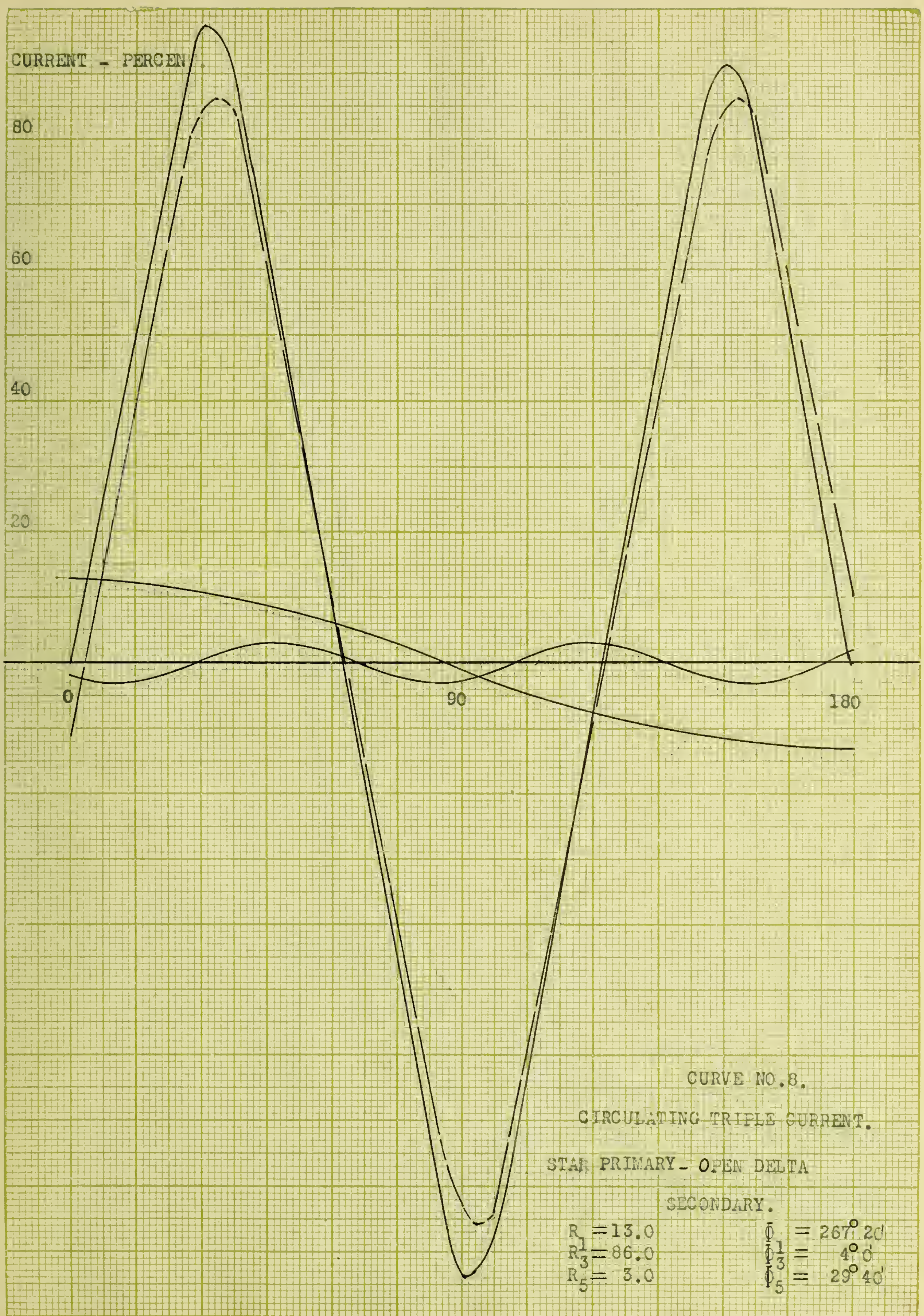


CURVE NO.7.

COIL VOLTAGE .

STAR PRIMARY - OPEN DELTA SECONDARY.

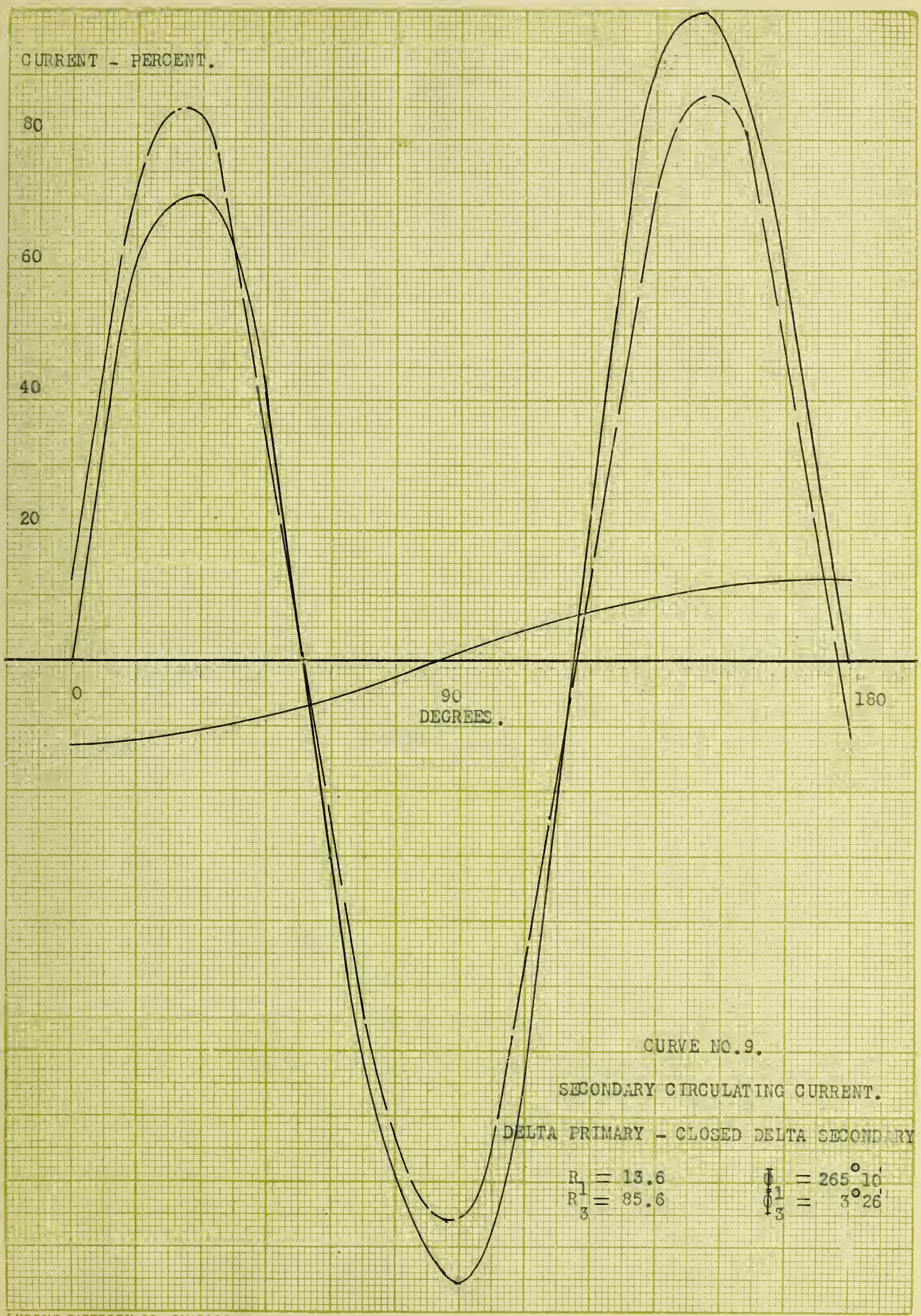
$R_1 = 65.$	$\Phi_1 = 4^\circ$
$R_3 = 38.1$	$\Phi_3 = 61^\circ$
$R_9 = 4.5$	$\Phi_9 = 62^\circ\ 30'$



CURVE NO. 8.

CIRCULATING TRIPLE CURRENT.

STAR PRIMARY – OPEN DELTA SECONDARY.



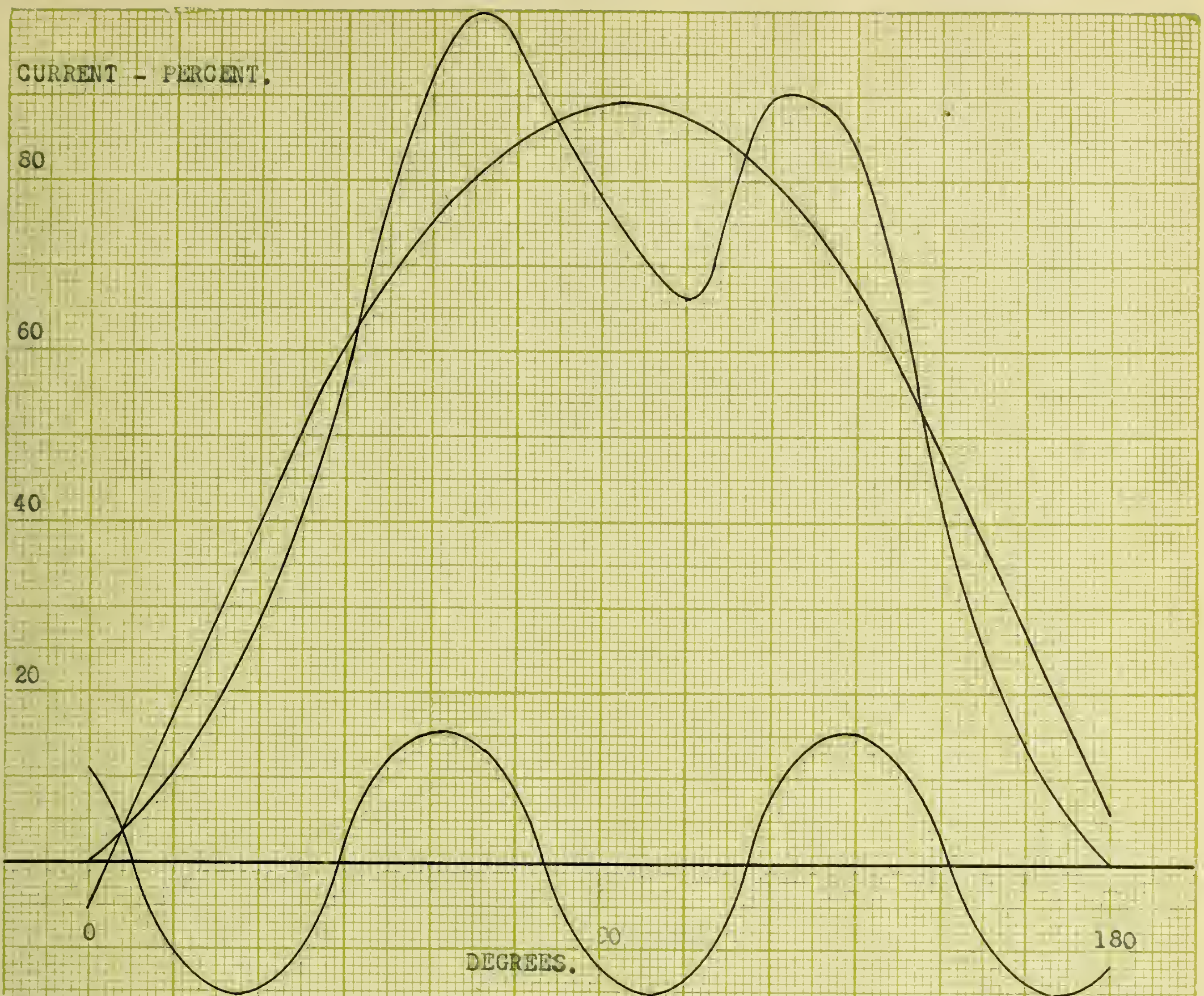
CURVE NO. 9.

SECONDARY CIRCULATING CURRENT.

DELTA PRIMARY - CLOSED DELTA SECONDARY

$R_1 = 13.6$ $\phi_1 = 265^\circ 10'$

$R_3 = 85.6$ $\phi_3 = 3^\circ 26'$



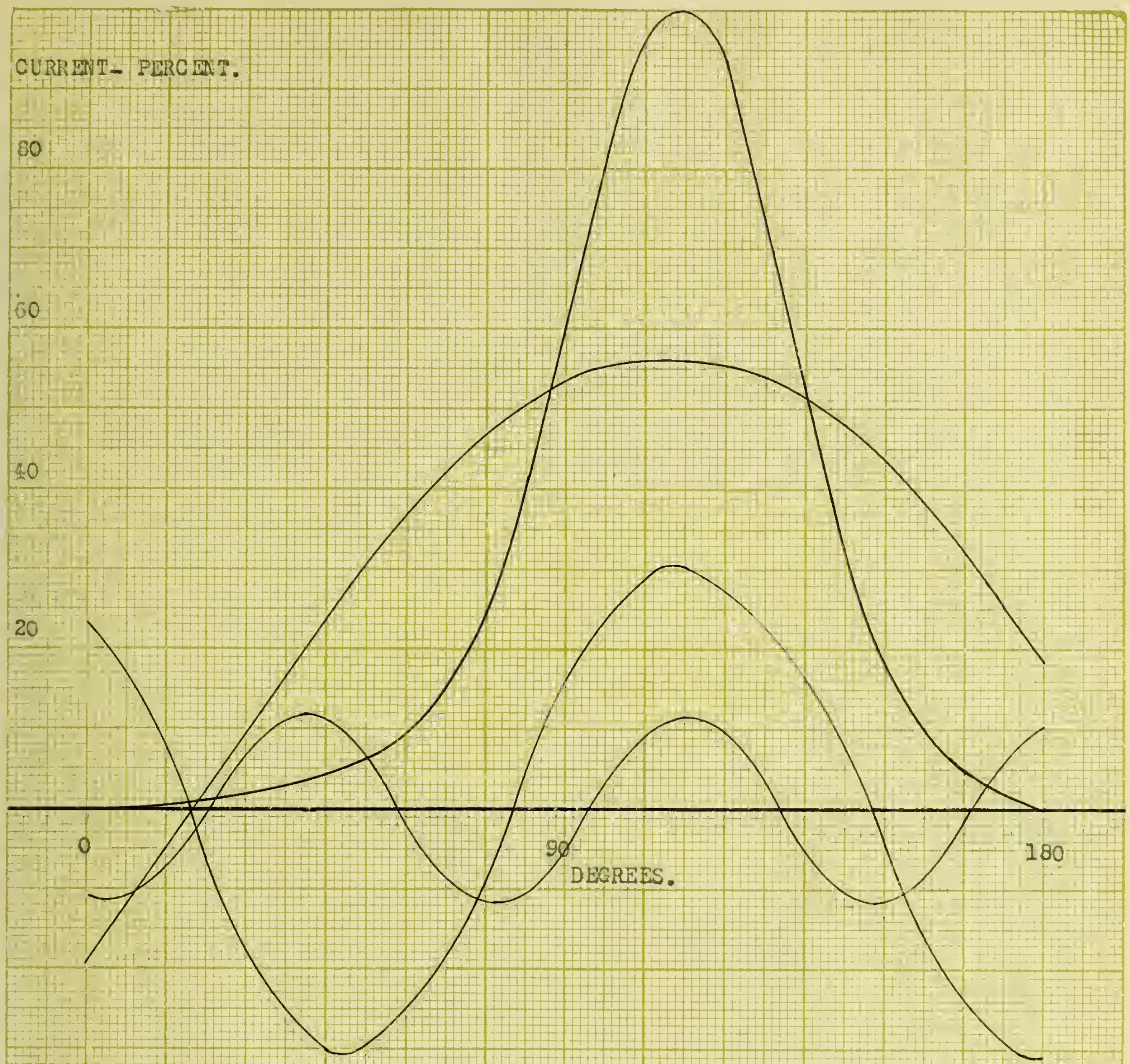
CURVE NO. 10.

PRIMARY LINE CURRENT.

DELTA PRIMARY- OPEN DELTA SECONDARY.

$R_1 = 89.2$ $\phi_1 = 3^\circ\ 3'$

$R_5 = 15.2$ $\phi_5 = 44^\circ\ 30'$



CURVE NO.II.

PRIMARY COIL CURRENT AND HARMONICS.

DELTA PRIMARY - OPEN DELTA SECONDARY.

$R_1 = 55.5$ $\qquad \phi_1 = 19^\circ 40'$

$R_3 = 30.8$ $\qquad \phi_3 = 80^\circ 40'$

$R_5 = 11.8$ $\qquad \phi_5 = 94^\circ 25'$